Effects of ΣN channels on the Spin-Orbit Force in Λ-hypernuclei

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Abstract: Effects of the ΣN channels on the spin-orbit force in Λ-hypernuclei are investigated. It is found that the cancellation between the symmetric and the antisymmetric ΛN spin-orbit force becomes more complete when the contribution from the ΣN channels is included. This may be the one of the reasons why the observed LS splitting of the Λ single particle energy is very small.

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The spin-orbit (LS) force in the two-baryon interaction has been investigated extensively. Information on the two-nucleon (NN) interaction can be directly obtained from the NN scattering observables. As is well-known, all the available phase shift analyses have concluded that there is a strong spin-orbit force between nucleons[1,2]. The LS force in the hyperon-nucleon (YN) interaction, however, is not well-investigated by the direct scattering experiments. From the observed single particle energy levels of Λ-hypernuclei, it is considered that the spin-orbit force between Λ and the nucleon is very small comparing to that between two nucleons[3,4,5].

The spin-orbit force of the YN interaction can be divided into the symmetric and the antisymmetric ones (SLS and ALS):

$$\mathcal{O}_{\substack{SLS \\ ALS}} = \frac{\sigma_i \pm \sigma_j}{2} \cdot i[\boldsymbol{r}_{ij} \times \boldsymbol{p}_{ij}] \,, \tag{1}$$

where $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$ is the relative coordinate of the baryons and $\boldsymbol{p}_{ij} = (m_j\boldsymbol{p}_i - m_i\boldsymbol{p}_j)/(m_i + m_j)$. ALS becomes important in systems with strangeness. The main part of ALS comes from the (symmetric) spin-orbit force in the quark-quark interaction and from the F/D difference of the vector-meson exchange, both of which survive at the flavor SU(3) limit[6,7]. The size of ALS can be comparable to the size of SLS though it depends strongly on the channels.

From a quark-model viewpoint, the size of SLS between Λ and the nucleon can be comparable to that between two nucleons. The spin-orbit force of the Λ particle, however, becomes small due to the SLS and ALS cancellation. This is because the s-quark alone among the quarks in the Λ particle produces the ΛN spin-orbit force: the combined spin of the u- and d-quark in the Λ particle is zero. This small LS is divided artificially into SLS and ALS, which cancel each other[7,8].

The ALS and SLS cancellation in the ΛN channels, however, may not be enough to explain the observed small LS splitting of the Λ single particle energy in hypernuclei. In ref.[9] several reasons why the LS splitting is small are discussed. In this letter, we show that the coupling to the ΣN channels also affects the LS splitting of Λ-hypernuclei largely; effective LS arising from the coupling to the ΣN channels reduces the splitting of the Λ single particle energy so that it becomes very small.

The model we employ here to investigate the above matter is the quark cluster model (QCM). This model with the instanton effects can explain the spin-orbit nature both in the two-baryon systems and in the single baryon mass spectrum from the same quark hamiltonian, and therefore enables us to investigate the spin-orbit force from more fundamental

viewpoints[7,8,10].

The present quark model contains four terms in the hamiltonian: the kinetic term, K_q, the confinement term, V_{conf}, and the one-gluon exchange (OGE) term, V_{OGE}, and the instanton-induced interaction (III) term V_{III}, viz.[7,10,11],

$$H_{\mathrm{quark}} = K_q + (1 - p_{\mathrm{III}})V_{\mathrm{OGE}} + p_{\mathrm{III}}V_{\mathrm{III}} + V_{\mathrm{conf}} . \tag{2}$$

The parameter p_{III} represents the relative strength of the spin-spin part of III to OGE. It corresponds to the rate of the contribution from III to the S-wave N-Δ mass difference, $\delta M_{\mathrm{N\Delta}}$.

The YN interaction can be obtained from QCM as follows[7,12,13,14]. The wave function is restricted as

$$\Psi_{6q} = \mathcal{A}_q\{\phi_B\phi_{B'}\chi(\boldsymbol{R})\} . \tag{3}$$

where $\mathcal{A}_q$ is an antisymmetrizing operator for quarks, ϕ_B is the wave function of the baryon B, and $\boldsymbol{R} = (\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3 - \boldsymbol{r}_4 - \boldsymbol{r}_5 - \boldsymbol{r}_6)/3$. By integrating out the internal coordinates, we have the RGM equation,

$$(H - EN)\chi = 0 , \tag{4}$$

where H is the hamiltonian kernel and N is the normalization kernel, both of which are nonlocal. The above equation can be rewritten as

$$(\overline{H} - E)\overline{\chi} = 0 \tag{5}$$

with

$$\overline{H} = N^{-1/2}HN^{-1/2} \tag{6}$$

$$\overline{\chi} = N^{1/2}\chi . \tag{7}$$

Thus, the short range part of the YN potential can be defined by

$$V_{\mathrm{QCM}} = \overline{H} - K_0 , \tag{8}$$

where K_0 is the kinetic term with $M_B = \sum m_q$. The realistic YN hamiltonian can be obtained as

$$H_{\mathrm{YN}} = K + V_{\mathrm{QCM}} + V_{\mathrm{mesons}} , \tag{9}$$

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where K is the kinetic term with the observed baryon masses, and V_{mesons} is the one-boson exchange potential with an appropriate form factor[7]. This hamiltonian can be used in usual Schrödinger equation:

$$(H_{\text{YN}} - E)\psi \;=\; 0 \;, \tag{10}$$

with $\int |\psi|^2 = 1$.

When there are one or more forbidden states due to the quark Pauli-blocking, the normalization kernel does not have an inverse operator. One has to remove the forbidden states from the wave function ψ in eq. (10) so that the operator $N^{-1/2}$ in eq. (6) becomes well-defined. The YN ($I{=}1/2$ $S{=}{-}1$) P-wave channel in the present issue, however, does not have such a state, and is free from the above difficulty when only the ΛN and ΣN channels are considered. Eq. (10) is equivalent to eq. (4) except for V_{mesons} and the choice of the baryon masses in the kinetic term.

The ΛN interaction which includes the effects of the ΣN channels is obtained as:

$$\tilde{H}_{\Lambda\text{N}}(E_0) \;=\; H_{11} - H_{12}(H_{22} - E_0)^{-1}H_{21} \;. \tag{11}$$

Here, H_{ij} is the (i, j)-component of $H_{\text{YN}} = \{H_{ij}\}$ in eq. (9), where the channel 1 [2] denotes the ΛN [ΣN] channel. The solution of the equation, $\{\tilde{H}_{\Lambda\text{N}}(E_0) - E\}\psi = 0$, is the same as that of the coupled channel equation when E_0 equals to E; the obtained scattering phase shifts are the same at $E = E_0$. The hamiltonian of the single channel calculation, $H_{\Lambda\text{N}}$, is obtained by following the same procedure from eq. (3) to eq. (9) with the space restricted to the ΛN channels. The difference, $\delta H \equiv \tilde{H}_{\Lambda\text{N}} - H_{\Lambda\text{N}}$, is considered to be the correction due to the effects of the ΣN channels.

We use the parameter sets QCM-B, -C and -D, which were reported in ref.[7], to investigate the issue. The QCM-B and -C contains no instanton effects ($p_{\text{III}}{=}0$). On the other hand, p_{III} is equal to 0.4 in QCM-D, which is consistent to the value that gives the observed η-η' mass difference and that gives the small LS splitting in the single baryon mass spectrum. All the parameter sets contain the flavor singlet and octet, scalar- pseudoscalar- and vector-meson exchange in V_{mesons}. Parameters in the quark part and in the meson part are determined so that the model reproduces the NN scattering data and the cross section of the Λp elastic scattering in the low-energy region. The parameter sets used in the meson-exchange part of QCM-C and -D are the best fit each for $p_{\text{III}} = 0$ and for $p_{\text{III}} = 0.4$, respectively. The coupling constants of the vector mesons in QCM-B are kept to the same values as those in Nijmegen SC97f though their strength is different from the original one due to the difference of the cut-off method.
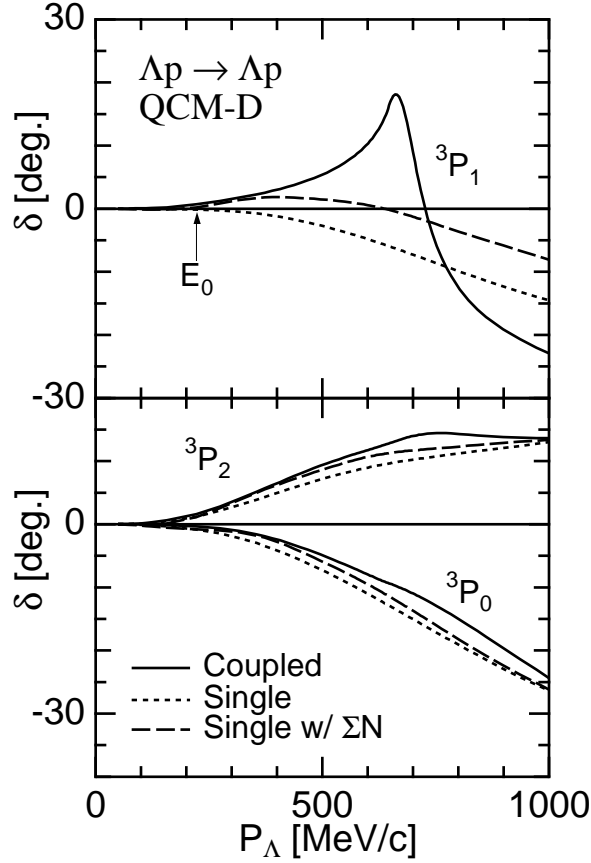
Figure 1: Phase shifts of the ΛN P-wave channels.
The solid lines are for the ΛN-ΣN coupled-channel calculation, the dotted lines for the ΛN single-channel calculation, the dashed lines for the single-channel calculation with $\tilde{H}_{\Lambda N}(E_0=10\mathrm{MeV})$ (see text).

In fig. 1, we plot the phase shifts for the ΛN ^3P_J channels given by QCM-D. The solid lines correspond to the ΛN-ΣN coupled-channel calculation. The phase shift of the $^3P_1$ channel, where the mixing of ΛN $^1P_1$ and ΣN $^3P_1$ channels by the ALS force becomes important, oscillates largely at the ΣN threshold. The dotted lines correspond to the single-channel calculation with $H_{\Lambda N}$. The dashed lines correspond to the results of single-channel calculation by $\tilde{H}_{\Lambda N}$ with $E_0 = 10$ MeV; the effective hamiltonian reproduces the coupled-channel results well except for the threshold behavior.

To see rough size of the effects of the ΣN channels on the Λ hypernuclei, we evaluate the spin-orbit part of the hamiltonian. The matrix elements of SLS and ALS each for $H_{\Lambda N}$ and $\delta H_{\Lambda N}$ are listed in table 1 together with the total LS, $\tilde{H}_{\Lambda NLS}$. The wave functions used for this estimate are (i) the harmonic oscillator wave function with the size parameter of $b_B =$ 1.35 [fm], (ii) the QCM wave function with the rms which corresponds to the one with $b_B =$

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Table 1: Evaluation of LS components

	QCM-B			QCM-C			QCM-D		
	(i)	(ii)	(iii)	(i)	(ii)	(iii)	(i)	(ii)	(iii)
$H_{\Lambda N SLS}$	-1.37	-1.03 (-0.56)	-0.50	-1.62	-1.22	-0.59	-1.48	-1.20	-0.58
$H_{\Lambda N ALS}$	0.45	0.31 (0.07)	0.15	0.41	0.26	0.12	0.30	0.22	0.10
$H_{\Lambda N LS}$	-0.91	-0.72 (-0.48)	-0.35	-1.21	-0.95	-0.46	-1.18	-0.98	-0.48
δH_{SLS}	0.65	0.58 (0.50)	0.27	0.62	0.56	0.26	0.59	0.54	0.25
δH_{ALS}	0.88	0.67 (0.07)	0.27	0.67	0.50	0.20	0.58	0.45	0.18
$\tilde{H}_{\Lambda N LS}$	0.62	0.53 (0.10)	0.19	0.08	0.10	0.00	-0.01	0.01	-0.04

1.35 fm, and (iii) that of 1.6 fm. To obtain the wave functions of (ii) and (iii), we solve the equation with an extra gaussian potential with the central part of $H_{\Lambda N}$,

$$\{H_{\Lambda N, cent} + U_0 R^2 - E\}\psi = 0 . \tag{12}$$

To see the effects of the difference in the hamiltonian clearly, the orbital part of the obtained wave function for the ^3P channel is used as both of that for the ^3P_J and for the $^1P_1$ channels. The value of U_0 is taken so that $\langle R^2 \rangle = 5b_B^2$ with $b_B = 1.35$ fm or 1.6 fm. This term also appears in the denominator of the second term of $\tilde{H}_{\Lambda N}$. By removing the term $U_0 R^2$ from the denominator, the correction listed in the table becomes larger by about $10\% \sim 20\%$. The eigen energy of eq. (12) is used for E_0 in $\tilde{H}_{\Lambda N}(E_0)$; when the eigenvalue of each channel is used as E_0, the results change by less than 1% in the present case.

It is found that the introduction of the ΣN channels reduces SLS and enhances ALS. As a result, the total LS, $\tilde{H}_{\Lambda N LS}$, which corresponds to the Λ LS splitting, becomes very small both in QCM-C and -D. Let us emphasize that the small $\tilde{H}_{\Lambda N LS}$ is preferred by the experiments.

It is interesting to see whether this mechanism of cancellation due to the ΣN channels only occurs in a quark model. We have not investigated the issue by using the meson-exchange models. For the reference, however, the results with no quark spin-orbit force are listed in parentheses for the QCM-B (ii) case in table 1. Though the absolute value is different, especially for ALS, which is weakened by the cut off, the fact that the effects of the ΣN channels reduce the total LS seems valid for the meson LS.

The correction can be written as

$$\delta H = (H_{11} - H_{\Lambda N}) + H_{12}(H_{22} - E_0)^{-1} H_{21} . \tag{13}$$

The first term of rhs is the contribution of the ΣN channels through the normalization kernel, which does not appear in the meson-exchange models. Its contribution, however,

seems minor: it enhances SLS only by less than 5%. This term for ALS is -0.09 for the case (ii) of QCM-D, for example; it is reverse and still considerably smaller comparing to the second term.

The second term of eq. (13), which is common to the meson-exchange models, gives a major contribution to the correction, δH. This contribution to ALS can be divided into four terms. Namely, each pair of (S, S') in

$$\sum_{S,S'} \langle \Lambda N^3 P_1 | H_{12} | S \rangle \langle S | (H_{22} - E_0)^{-1} | S' \rangle \langle S' | H_{21} | \Lambda N^1 P_1 \rangle \ . \tag{14}$$

where $|S\rangle$ stands for $|\Sigma N^{2S+1} P_1\rangle$. It is found that all the four terms contribute additively to δH_{ALS}. The channel dependence of ALS is determined only by the flavor SU(3) symmetry and does not depend on the origins of ALS[6]. That is, the strength of ALS in a certain channel is essentially governed by the symmetry and one dynamical matrix element. One the other hand, the factor $\langle S | (H_{22} - E_0)^{-1} | S' \rangle$ should not change much if one uses a realistic YN potential. Therefore, ALS of the system should enhance similarly by eq. (14), provided that the ΛN-ΣN off-diagonal matrix element given by QCM is similar to the one with the meson-exchange potentials; which is likely because the models maintain the flavor SU(3) symmetry approximately.

The situation for SLS is vague, because the channel dependence of SLS is not determined only by the symmetry unlike that of ALS. The sign of meson SLS, however, is the same as that of SLS originated from OGE or III interacting between quarks except for the ΣN-ΣN $(I = 1/2)$ ^3P_J channel[6]. Therefore, we expect that the similar correction arises also for SLS.

As is seen in table 1, the size of the effects may vary. Here, at least for the best fit models which reproduces the NN scattering data and the ΛN low energy data, it seems the LS becomes almost zero by introducing the present effects. Recently, a work investigating Λ LS in hypernuclei by using the G-matrix approach with a quark cluster model has been reported[15]; they have concluded that the small LS splitting for Λ in hypernuclei can be obtained from their model. The similar mechanism to the present work probably occurs in their approach.

In this letter we investigate the effects of the coupling to the ΣN channels on the ΛN spin-orbit force employing the quark cluster model. It is found that the coupling reduces the LS force considerably. This mechanism probably contributes to the observed small LS splitting of the Λ single particle energy.

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